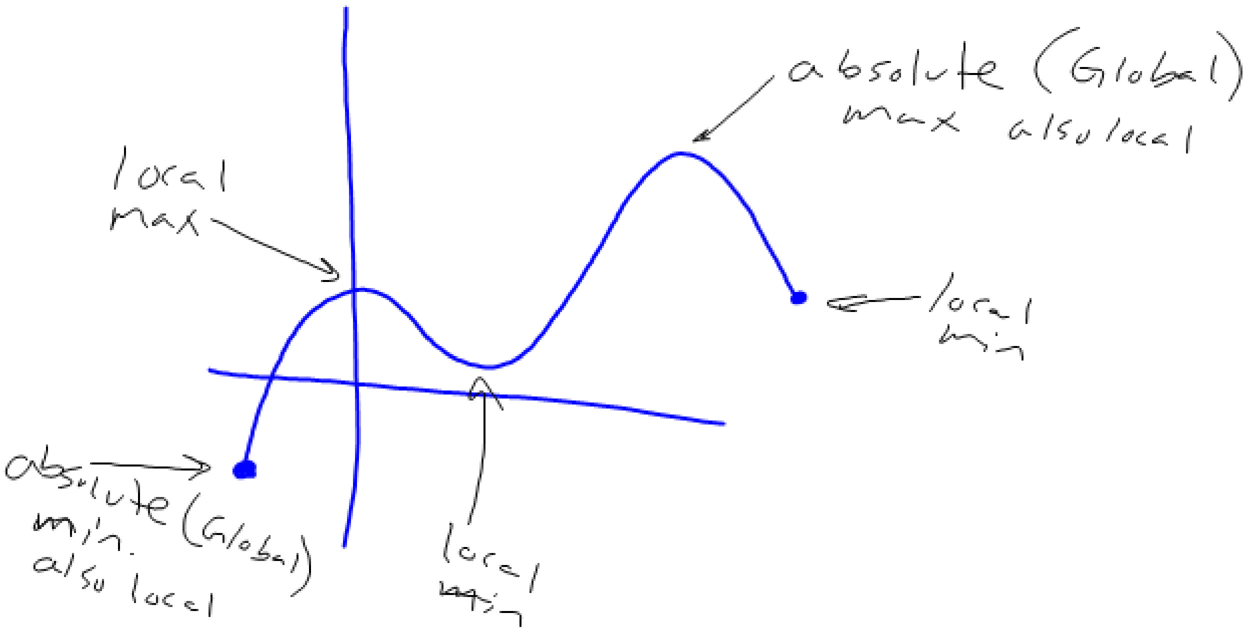
extrema - max, min



Absolute max at $x=c$ if $f(x) \leq f(c)$ for all x in Domain

Absolute min at $x=c$ if $f(x) \geq f(c)$ for all x in Domain

Local max at $x=c$ if $f(x) \leq f(c)$ for all x in some open interval with c as an interior point of domain

Local min at $x=c$ if $f(x) \geq f(c)$ for all ....

Extreme Value Thrm

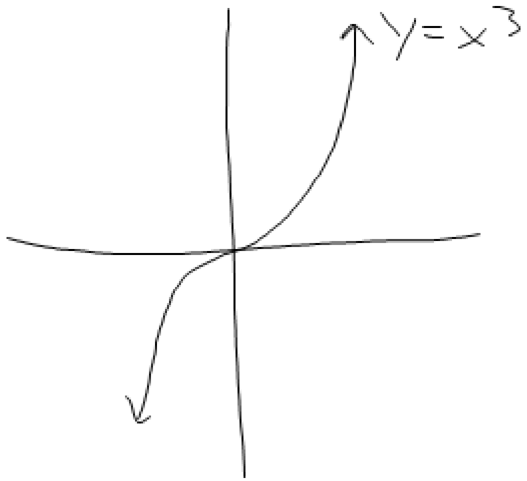
THEOREM 1 The Extreme Value Theorem

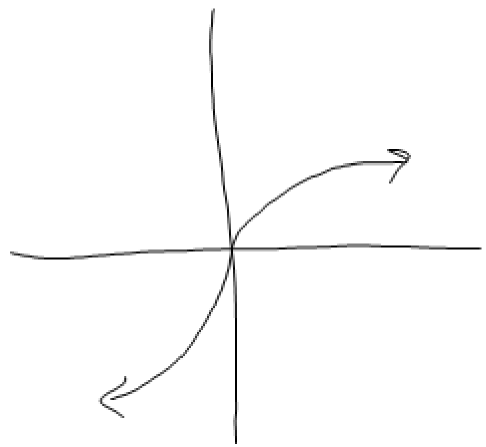
If f is continuous on a closed interval $[a, b]$, then f has both a maximum value and a minimum value on the interval. (Figure 4.3)

Critical Point

A point in the interior at which $f'(x) = 0$ or does not exist

that is where the extrema may occur and the end points





Confirm Analytically The function f is defined only for $4 - x^2 > 0$, so its domain is the open interval $(-2, 2)$. The domain has no endpoints, so all the extreme values must occur at critical points. We rewrite the formula for f to find f':

$$f(x) = \frac{1}{\sqrt{4 - x^2}} = (4 - x^2)^{-1/2}.$$

Thus,

$$f'(x) = -\frac{1}{2}(4 - x^2)^{-3/2}(-2x) = \frac{x}{(4 - x^2)^{3/2}}. = 0 \rightarrow x = 0$$

The only critical point in the domain $(-2, 2)$ is $x = 0$. The value undef. at $x = -2, 2 \rightarrow$ not in domain

$$f(0) = \frac{1}{\sqrt{4 - 0^2}} = \frac{1}{2}$$

is therefore the sole candidate for an extreme value.

To determine whether $1/2$ is an extreme value of f, we examine the formula

$$f(x) = \frac{1}{\sqrt{4 - x^2}}.$$

As x moves away from 0 on either side, the denominator gets smaller, the values of f increase, and the graph rises. We have a minimum value at $x = 0$, and the minimum is absolute.

The function has no maxima, either local or absolute. This does not violate Theorem 1 (The Extreme Value Theorem) because here f is defined on an *open* interval. To invoke Theorem 1's guarantee of extreme points, the interval must be closed.

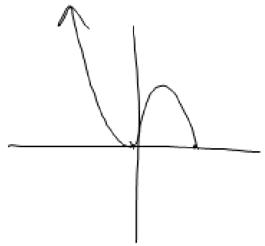
$y = x^2\sqrt{3-x}$ find critical values

Domain

$(-\infty, 3]$

$$y' = x^2 \cdot \frac{1}{2\sqrt{3-x}}(-1) + \sqrt{3-x}\cdot 2x$$

$$y' = \frac{-x^2}{2\sqrt{3-x}} + \frac{2x\sqrt{3-x}}{1}\cdot\frac{2\sqrt{3-x}}{2\sqrt{3-x}}$$



$$y' = \frac{-x^2+4x(3-x)}{2\sqrt{3-x}} \Rightarrow \frac{-5x^2+12x}{2\sqrt{3-x}}$$

$$\cancel{2\sqrt{3-x}}\;\frac{-5x^2+12x}{\cancel{2\sqrt{3-x}}} = 0\cdot 2\sqrt{3-x}$$

$$-5x^2+12x=0$$

$$(x)(-5x+12)=0$$

$$\boxed{x=0,\ \frac{12}{5}}$$

undefined

$$\boxed{x=3}$$

$f(0)=0$ min

$f(3)=0$ min

$f(\frac{12}{5})\approx 4.462$ local max

$f(-100)\approx$ way big

Sect. 4.1

#8, 10, 16, 18, 19, 22, 23, 29, 30, 37, 40, 42